"Our school community of Sacred Heart reflects the love that Jesus has for us. This creates a successful learning environment where we welcome people so they feel happy, settled and included. We honour each other and our school with pride."

MATHEMATICS CALCULATION POLICY SEPTEMBER 2016

About Our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Foundation Stage follows the 'Development Matters' EYFS document and this calculation policy is designed to build on progressively from the methods and content established in the Early Years Foundation Stage.

Age Stage Expectations

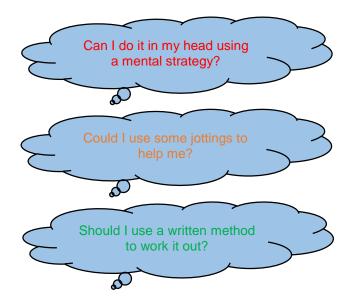
The calculation policy is organised according to age stage expectations set out in the National Curriculum 2014; however it is vital that pupils are taught according the stage at which they are currently working, being moved on to the next level as soon as they are ready or working at a lower stage until they are secure enough to move on.

Providing a context for calculation

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation to ensure they select the most appropriate method for the numbers involved:



To work out a tricky calculation

Approximate,

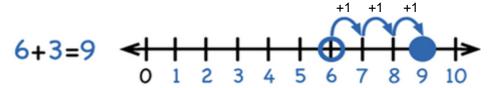
Calculate,

Check it mate!

Addition

Year 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with larger numbers and count on.



Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Solve one step problems involving addition.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through tens by counting on 2 then counting on 3.

8 + 5

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, put together

Key skills for addition at Y1:

- Read and write numbers to 100 in numerals, including 1 20 in words
- Recall bonds to 10 and 20 and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations

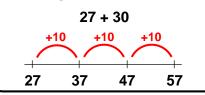
Video clip: Using a range of equipment and strategies to reinforce addition statements

Year 2 Add with 2-digit numbers

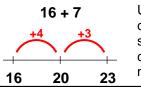


Developing mental fluency with addition and place value involving 2-digit numbers; then establishing more formal methods.

Add 2-digit numbers and tens:



Add 2-digit numbers and units:



3

= 7 9

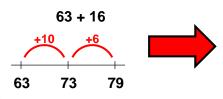
Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units.

6 0 +

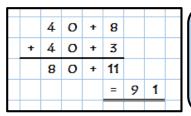
1 0 + 6

7 0 + 9



STEP 1: Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80 + 11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 48 + 43).



STEP 3: Children who are accurate and confident with this stage should move onto the expanded addition methods with 2- and 3-digit numbers (see Y3).

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters; then compare their practical version to the written form to help them build an understanding of it.

<u>Key Vocabulary:</u> add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, put together, sum, tens, units, partition, addition, column, tens boundary

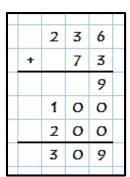
Key skills for addition at Y2:

- Add a 2-digit number and units (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 7 + 9)
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number
- Understand the place value of 2-digit numbers (tens and units)
- Compare and order numbers to 100 using <, > and = signs
- Read and write numbers to 100 in numerals and words
- Solve problems with addition using concrete objects and pictorial representations involving numbers, quantities and measures and applying mental and written methods



Year 3 Add numbers with up to 3 digits

Introduce the expanded column addition method:



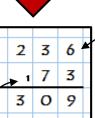
Add the units first, in preparation for the formal method.

In order to carry out this method of addition:

- Children need to recognise the place value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.



Move to the formal column method, with 'carrying':



Add units first.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the formal column method, being introduced to 'carrying' for the first time. Compare the expanded method to the formal column method to develop an understanding of the process and the reduced number of steps involved.

Carry numbers on the top line.

Remind pupils that the actual value is 'three tens add seven tens', not 'three add seven', which equals ten tens.

Key Vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, put together, sum, tens, units, partition, addition, column, tens boundary, hundreds, hundreds boundary, increase, vertical, 'carry', expanded, formal

Key skills for addition at Y3:

- Read and write numbers to 1,000 in numerals and words
- Add 2-digit numbers mentally, including those with answers exceeding 100
- Add a 3-digit number and units mentally (e.g. 175 + 8)
- Add a 3-digit number and units mentally (e.g. 249 + 50)
- Add a 3-digit number and hundreds mentally (e.g. 381 + 400)
- Estimate answers to calculations, using inverse to check answers
- Solve problems, including missing number problems, using number facts, place value and more complex addition
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, units)
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding the nearest multiple of 10, 100, 1000 and adjusting, using near doubles, partitioning and recombining

Video clip: Demonstration of expanded 3-digit column addition

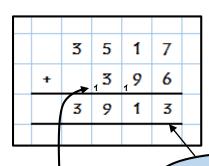
Addition

Year 4 Add numbers with up to 4 digits



Move from expanded addition to the formal column method, **adding units first**, and 'carrying' numbers above the answer and below the calculation. Also include money and measures contexts.

$$e.g. 3517 + 396 = 3913$$



Introduce the formal column addition method by asking children to add the two given numbers together using the method with which they are familiar (expanded column addition – see Y3). Teacher models the formal method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add units first.

'Carry' numbers above the first line.

Reinforce correct place value by reminding children the actual value is **5 hundreds** add **3 hundreds**, not 5 add 3, for example.

Use and apply this method to money and measurement values.

<u>Key Vocabulary:</u> add, more, plus, and, make, altogether, total, equals to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds, hundreds boundary, increase, vertical, 'carry', expanded, formal, tenths, hundredths, decimal point, thousands, digits, inverse

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why
- Recognise the place value of each digit in a four-digit number
- Round any number to the nearest 10, 100 or 1000
- Estimate and use inverse operations to check answers
- Solve 2-step problems in context, deciding which operations and methods to use and why
- Find 1000 more or less than a given number
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, using near doubles, partitioning and recombining
- Add numbers with up to four digits using the formal written method of column addition
- Estimate and use inverse operations to check answers to a calculation

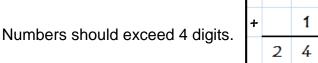


Year 5 Add numbers with more than 4 digits

This will include money, measures and decimals with different numbers of decimal places.



The decimal point should be placed on the line between the units and the tenths columns and aligned so that each decimal point is directly below the previous one.



	2	3	4	8	1	
+		1	. <mark>3</mark>	6	2	
	2	4	8	4	3	

	1	9	О	1	
+		3	6	5	
+	1	, <mark>O</mark> .	7	0	_
	2	3 .	3	6	

Pupils should be able to add more than two values, carefully aligning place value columns.

Encourage children to fill in the 'missing' zero in empty decimal places.

Say '6 tenths add 7 tenths' to reinforce place value.

Children should:

 Understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.

<u>Key Vocabulary:</u> add, more, plus, and, make, altogether, total, equals to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, formal, tenths, hundredths, decimal point, thousands, hundreds, digits, inverse, decimal places, thousandths

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. add the nearest multiple of 10, 100, 1000 and adjust; using near doubles, inverse, partitioning and re-combining; using number bonds
- Use rounding to check answers and accuracy
- Solve multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least one million and determine the value of each digit
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10 000 and 100 000
- Add numbers with more than four digits using the formal written method of column addition

Year 6 Add several numbers of increasing complexity



	2	3 .	3	6	1	
+		9 .	O	8	O	
+	5	9 .	7	7	O	
+	2	, 1 ,	₂ 3	O	0	t
	9	3 .	5	1	1	

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer
- Write a zero where there are empty decimal places to show there is no value to add

Put a zero in empty decimal places to show the place value in each column.

	8	1	0	5	9	
+		3	6	6	8	
+	1	5	3	O	1	
+	₁ 2	1 <mark>0</mark>	₁ 5	₁ 5	1	
1	2	0	5	7	9	

Adding several numbers with more than 4 digits.

Key Vocabulary: add, more, plus, and, make, altogether, total, equals to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, formal, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies
- Solve multi-step problems in context, deciding which operations to use and why
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity



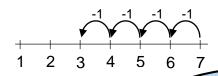
Year 1 Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using numbers lines as below.

Read, write and interpret number sentences with – and = signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20.



7 - 4 = 3

Model subtraction using hundred squares and numbered number lines or tracks and practically.

Find the 'distance between'

This will be introduced practically with the language 'find the distance between' and 'how many more?' in a range of familiar contexts.

000000

7 'Seven is 3 more than 4'

'I am 2 years older than my sister'

Mental subtraction

Children should start recalling facts up to and within 10 and 20 and should be able to subtract zero.

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, tens, units, count back, how many left, how much less is...?

Key skills for subtraction at Y1:

- Given a number, say one more or one less
- Count to and over 100, **forward and back**, from any number
- Represent and use subtraction facts to 20 and within 20
- Subtract with one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems
- Read and write numbers from 0 to 20 in numerals and words

Year 2 Subtract with 2-digit numbers

Subtract on a number line by **counting back**, aiming to develop mental subtraction skills.

Use Dienes blocks for subtraction calculations too.

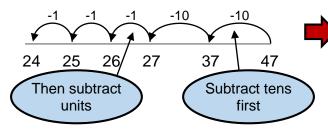
This strategy will be used for:

- 2-digit numbers subtract units (by taking away / counting back) e.g. 36 7
- 2-digit numbers subtract tens (by taking away / counting back) e.g. 48 30
- Subtracting pairs of 2-digit numbers (see below)

Subtracting pairs of 2-digit numbers on a number line:

47 - 23 = 24 Partition the second number and subtract it in tens and units as below:

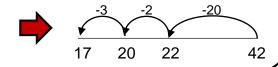
Move towards more efficient jumps back as below:



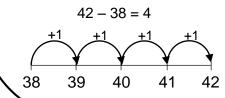


Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42 – 25:



Mental strategy - subtract numbers close together by counting on:



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, tens, units, count back, how many left, how much less is...?, difference, count on, strategy, partition,

Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number
- Recall and use subtraction facts to 20 fluently and derive and use related facts up to 100
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including a 2-digit number and units, a 2-digit number and tens and two 2-digit numbers
- Show that subtraction of one number from another cannot be done in any order
- Recognise and use inverse relationships between addition and subtraction, using this to check calculations and missing number problems
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representations and also applying their increasing knowledge of mental and written methods
- Read and write numbers to at least 100 in numerals and words

Subtraction

Year 3 Subtract with 2-digit and 3-digit numbers

Introduce partitioned column subtraction method.

<u>Step 1</u>: Introduce this method with examples where no exchanging is needed.

$$89 - 35 = 54$$

$$80 + 9$$

$$- 30 + 5$$

$$50 + 4$$

When learning to 'exchange', explore partitioning in different ways so that pupils understand that when you exchange, the **value** is the same i.e. 72 = 70 + 2 = 60 + 12 etc. Emphasize that the **value hasn't changed**, we have just partitioned in a different way.

<u>Step 2</u>: Introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract from it.



Before subtracting 7 from the unit blocks, they will need to exchange a row of 10 for ten units. Then subtract 7 units and subtract 4 tens.

Step 3: Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2-digit and 3-digit numbers.

Subtracting money, partition into e.g. £1 + 30p + 8p

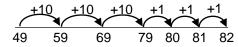
Video clips:

Clip 1 Clip 2

Counting on as a mental strategy for subtraction:

Continue to reinforce **counting on** as a strategy for close-together numbers (e.g. 121 - 118) and also for numbers that are nearly multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102 - 89, 131 - 79 or calculating change from £1 etc.).

Start at the smaller number and count on **in tens first**, then count on in units to find the rest of the difference:



<u>Key Vocabulary:</u> equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is...?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit

Key skills for subtraction at Y3:

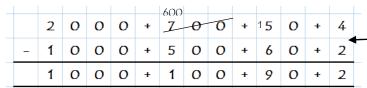
- Subtract mentally a 3-digit number and units, 3-digit number and tens, 3-digit number and hundreds
- Estimate answers and use inverse operations to check
- Solve problems, including missing number problems
- Find 10 and 100 more or less than a given number
- Recognise the place value of each digit in a 3-digit number
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers to 1000 in numerals and words
- Practise mental subtraction strategies such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21) and select most appropriate methods to use, explaining why



Year 4 Subtract with up to 4-digit numbers

Partitioned column subtraction with 'exchanging' (decomposition):

2754 - 1562 = 1192



As introduced in Y3, but moving towards more complex numbers and values. Use place value counters to reinforce 'exchanging'.



Formal column subtraction:

	2	67	15	4
_	1	5	6	2
	1	1	9	2

Give plenty of opportunities to apply this to money and measures

Subtracting money, partition into e.g. £1 + 30p + 8p

To introduce the formal method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the formal version for the calculation they have done. Ask pupils to consider how it relates to the method they know (what is similar, what is different) to develop an understanding of it.

Always encourage children to consider the best method for the numbers involved – mental, counting on, counting back or written method.

Mental strategies:

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on, e.g. 303 – 297.

<u>Key Vocabulary:</u> equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is...?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why
- Solve simple measure and money problems involving fractions and decimals to 2 d.p.
- Estimate and use inverse operations to check answers
- Find 1000 more or less than a given number
- Count backwards through zero, including negative numbers
- Recognise the place value of each digit in a 4-digit number.
- Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers

Video clips:

Clip 1 Clip 2 Clip 3



Year 5 Subtract with at least 4-digit numbers

This should include money, measures and decimals.

Formal column subtraction (with 'exchanging'):

	² 3	10 /	¹ O	⁴ 5	¹ 6
_		2	1	2	8
	2	8	9	2	8

Subtracting with larger integers.

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the formal method.

	⁶ ₹	10 X	¹ 6	8 9	¹ O	İ
_		3	7	2 ,	5	
	6	7	9	6 ,	, 5	Ī

Create lots of opportunities for subtracting and finding differences with money and measure.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Add a **zero** in any empty decimal places to aid understanding of what to subtract in that column.

Mental Strategies:

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on, e.g. 303 – 297, 2004 – 1997.

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is...?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal, decimal point, thousands

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000, and 100 000

Video Clip: Clip 1



Year 6 Subtracting with increasingly large and more complex numbers and decimal values

				,				
	_		8	9	9	4	9	
			6	O	7	5	O	
	°*	1 Ø	¹5 ,	³ <i>K</i>	1 1	9	kg	
-		3	6	. 0	8	0	kg	
		6	9	, 3	3	9	\kg	

Understand the formal column method to subtract more complex integers.

Using the formal column method to subtract money and measures, including decimals with different numbers of decimal places.

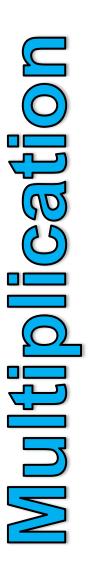
Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

Key Vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is...?, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal, decimal point

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context and calculate intervals across zero
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate



Year 1 Multiply with concrete objects, arrays and pictorial representations



How many legs will 3 teddies have?



2 + 2 + 2 = 6

This calculation shows 2 multiplied by 3, so $2 \times 3 = 6$.

There are 5 sweets in a bag. How many sweets are in 4 bags altogether?







5 + 5 + 5 + 5 = 20

This calculation shows 5 multiplied by 4, so $5 \times 4 = 20$.

- Give children experience of counting equal groups of objects in 2s, 5s and 10s
- Present practical problem solving activities involving counting equal sets or groups, as above



 $5 \times 2 = 10$



 $2 \times 5 = 10$

• Use arrays to represent multiplications, with the number of columns first and the number of rows second as in the example above

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, multiplied by, count

Key skills for multiplication at Y1:

- Count in multiples of 2, 5 and 10
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Make connections between arrays, number patters and counting in twos, fives and tens
- Begin to understand doubling using concrete objects and pictorial representations

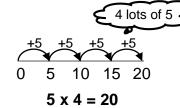
Multiplication

Year 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

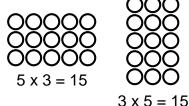


Use repeated addition on a number line:

 Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:



It is important to get these the correct way round.

 $5 \times 3 = 5 + 5 + 5 = 15$ $3 \times 5 = 3 + 3 + 3 + 3 + 3 = 15$

Use arrays to help teach children to understand the **commutative** law of multiplication.

Use missing number examples such as 3 x ___ = 6.

Use practical apparatus:

$$5 \times 3 = 5 + 5 + 5 = 15$$



Use mental recall:

• Children should be able to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understand of the operation.

<u>Key Vocabulary:</u> groups of, lots of, times, array, altogether, multiply, multiplied by, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times (etc.)

Key skills for multiplication at Y2:

- Count in steps of 2, 3 and 5 from zero and in 10s from any given number
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odds and evens
- Write and calculate number statements using the x and = signs
- Show that multiplication can be done in any order (commutative)
- Solve a range of problems involving multiplication using concrete objects, arrays, repeated addition, mental methods and multiplication facts
- Pupils use a variety of language to discuss and describe multiplication

Video Clips: Understanding multiplication facts

Practical multiplication and commutativity



Year 3 Multiply 2-digit numbers by a single digit number



Introduce the grid method for multiplying 2-digits by single digits:

Initially, link the layout of the grid to arrays, then move to just using numbers:

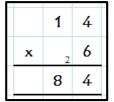
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to the grid method format (see video clip).

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables
- Work out multiplication facts not known by repeated addition or taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.). Strategies to support this are repeated addition using a number line, bead bars and arrays

Formal written method:

Children who are secure with the grid method can look to progress to the formal written method for multiplication (see Year 4)

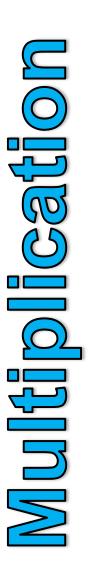


<u>Key Vocabulary:</u> groups of, lots of, times, array, altogether, multiply, multiplied by, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times (etc.), partition, grid method, formal method, multiple, product, tens, units, value, carry

Key skills for multiplication at Y3:

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables and multiply multiples of 10
- Write and calculate number statements using the multiplication tables they know, including 2-digit x 1-digit, drawing upon mental methods and progressing to reliable written methods
- Solve multiplication problems, including missing number problems
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in context, deciding which operations and methods to use
- Develop efficient mental methods (e.g. using commutativity) to solve missing number problems

Video Clip: Teaching the grid method as an interim step (using counters and partitioning)



Year 4 Multiply 2- and 3-digit numbers by a single digit (Using all multiplication tables up to 12 x 12)



Developing the formal method for multiplication:

Children should start off using the expanded formal method:

If needed, show how these numbers link to the grid method.

Carry above the line when adding. (See addition policy)

		1	6	7				İ
	x			6				
	/		4	2	(7	x 6)		
+		3	6	0	(60) x 6	() _V	
+	1	₁ 6	0	0	(10	00 x	6)	
	1	0	0	2				
								Τ

Like addition and subtraction, this method moves from right to left.

Emphasize that each digit is being multiplied by 3 by showing the partition at the side.

Children should then progress to the compact formal method:

As with addition, write the 'units' and carry the 'tens', i.e. for 42 write the 2 and carry the 4.

١		1	6	7
	x	4	4	6
	1	0	O	2

The steps should now be read as '7 x 6', '6 x 6' and '1 x 6' not '7 x 6', '60 x 6' and '100 x 6'. For example, the second step would be "6 x 6 is 36, plus the 4 is 40. Write the 0 and carry the 4."

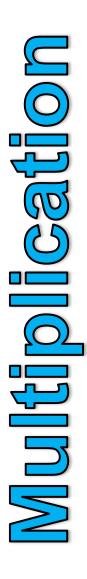
Children should be able to:

- Approximate before they calculate and make this a regular part of their calculating, going back to their approximation to check the reasonableness of their answer. The approximation should be recorded.
 E.g. 346 x 9 is approximately 350 x 10 = 3500
- Multiply multiples of 10 and 100 by a single digit using their multiplication table knowledge.

<u>Key Vocabulary:</u> groups of, lots of, times, array, altogether, multiply, multiplied by, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times (etc.), partition, grid method, formal method, multiple, product, tens, units, value, carry, expanded/compact formal method, inverse

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for all multiplication tables up to 12 x 12
- Recognise place value of digits in numbers up to 4 digits
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0 or to multiply 3 numbers together
- Use commutativity and other mental methods e.g. 2 x 6 x 5 = 10 x 6, 39 x 7 = 30 x 7 + 9 x 7
- Solve problems with increasingly complex multiplication in a range of contexts



Year 5 Multiply up to 4 digits by 1 or 2 digits



Introducing column multiplication for 2-digit multipliers:

- Children should understand that they are first multiplying their number (2, 3 or 4 digits) by the units of the multiplier and then by the tens number using the compact formal method they learned in Year 4
- Children should set out their calculation just as in column addition and subtraction making sure units and tens are aligned vertically

3 7 8 2 X, 1 5 1 2 + 5 6 Ο. 7 0 2

Once the smaller numbers have been added, cross them out to avoid confusion.

Numbers carried when adding go above this line as normal.

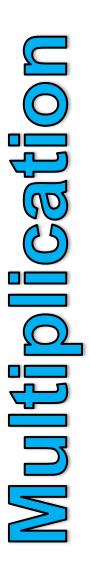
Emphasize the importance of the zero as a place holder as we are multiplying by tens now and not units.

- As with the compact formal method from year 4, each step should be read as a single digit by single digit multiplication. E.g. the last step in the above calculation would be 3 x 2 = 6 not 300 x 20 = 6000. This avoids confusion when adding on the 1 to ensure the answer of 7 not 6001
- Children should estimate before calculating to ensure the reasonableness of their answer. An estimate for this question could be 400 x 25 = 10,000 (as children should be comfortable finding multiples of 25)
- Children should start with 2-digit x 2-digit, then progress to 3-digit x 2-digit and then 4-digit x 2-digit. Establish that the process is identical regardless of the size of the top number.

<u>Key Vocabulary:</u> groups of, lots of, times, array, altogether, multiply, multiplied by, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times (etc.), partition, grid method, formal method, multiple, product, tens, units, value, carry, expanded/compact formal method, inverse, square, factor, integer, decimal

Key skills for multiplication at Y5:

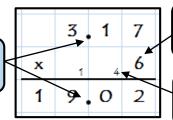
- Identify multiples and factors, using knowledge of multiplication tables up to 12 x 12
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and method appropriately



Year 6 Multiply decimals with up to 2 decimal places by a single digit



Decimals points are always aligned vertically.



Units go below the last digit of the top number.

Carried numbers above the line as before.

Children should be able to:

- Recognise that this is the only time when place value columns don't have to align. This is allowed because each digit in the top number is being multiplied by the bottom number
- Apply this method to multiplying with money
- Use estimation to check the reasonableness of their answer. E.g. for this question estimate using $3 \times 6 = 18$
- Use their estimate to recognise an incorrectly positioned decimal point in their answer. E.g. a child getting 190.2 for the example above

Key Vocabulary: groups of, lots of, times, array, altogether, multiply, multiplied by, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times (etc.), partition, grid method, formal method, multiple, product, tens, units, value, carry, expanded/compact formal method, inverse, square, factor, integer, decimal, tenths, hundredths, decimal

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to 12 x 12
- Multiply multi-digit numbers, up to 4-digit x 2-digit using the compact formal method
- Perform mental calculations with mixed operations and large numbers
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods
- Estimate answers using rounding and approximation and determine levels of accuracy
- Round any integer to a required degree of accuracy



Year 1 Group and share small quantities

Children should use objects, diagrams and pictorial representations to solve problems involving **both** grouping **and** sharing:

Grouping: How many groups of 4 can be made with 12 stars?







Answer = 3

Sharing: What is 10 shared between 2?





Answer = 5

Children should:

- Use lots of practical apparatus, arrays and pictorial representations
- Be taught to understand the difference between 'grouping' objects (e.g. how many groups of 2 can you make?) and sharing objects (e.g. share these sweets between 2 people.)
- Be able to count in multiples of 2, 5 and 10
- Find half of a group of objects by sharing into 2 equal groups
- Give answers using different vocabulary e.g. "18 shared between 6 people gives 3 each." and "you can make 3 groups of 6 from 18."

Key Vocabulary: share, share equally, one each, two each etc..., groups, groups of, array

Key skills for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Through grouping and sharing, pupils begin to understand division and finding simple fractions of objects, numbers and quantities
- Make connections between arrays, number patterns and counting in 2s, 5s and 10s

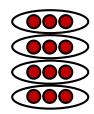


Year 2 Group and share using the ÷ and = signs



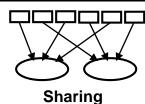
Children should use objects, arrays, diagrams and pictorial representations and grouping on a number line.

Arrays:



This array could represent $12 \div 3 = 4$ if the question was "How many groups of 3 can be made from 12?" or $12 \div 4 = 3$ if the question was "Share 12 equally between 4." Children should recognise both of these possibilities.

Know and understand sharing and grouping:



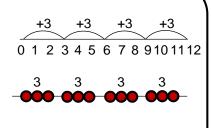


Grouping

Children should be taught to understand whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out how many groups of *x* in *y*. Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" This is an important method to develop understanding of division as grouping.

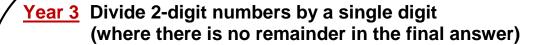


12 ÷ 3 = 4

<u>Key Vocabulary:</u> share, share equally, one each, two each etc..., groups, groups of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key skills for division at Y2:

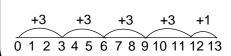
- Count in steps of 2,3 and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts





Grouping on a number line:

$$13 \div 3 = 4 r 1$$



Step 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s ready for 'carrying' remainders within the short division method.

Short Division:

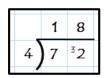
(with no remainders or carrying. i.e. each digit is a multiple of the divisor)



Step 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by comparing the layout of short division to arrays. Remind children of correct place value, i.e. 96 equals 9 tens and 6 units, but read the steps as: "How many 3s in 9?" and "How many 3s in 6?"

Short Division:

(with carrying but no remainders in the final answer)



Step 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $72 \div 4$) and be taught to 'carry' the remainder on to the next digit. **The next step should now be read as "How many 4s in 32?"**

If needed, children should use the number line to work out individual additions that they cannot recall mentally.

It is worth emphasizing that this is the only written method which goes 'left to right'!

Real life contexts need to be used routinely to help pupils gain a full
understanding and the ability to recognise the place of division and how
to apply it to problems.

Key Vocabulary: share, share equally, one each, two each etc..., groups, groups of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

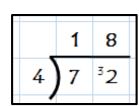
Key skills for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8, and 10 multiplication tables and, through doubling, connect the 2s, 4s and 8s
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for 2-digit numbers multiplied by 1-digit numbers, using mental methods and progressing to formal written methods
- Solve problems in contexts, including missing number problems involving multiplication and division
- Pupils develop effective mental methods, for example using multiplication and division facts to derive related facts
- Pupils develop reliable written methods for division using the progression outlined above

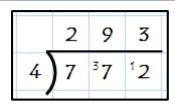
Year 4 Divide up to 3-digit numbers by a single digit



Continue to develop short division:



Step 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit that do not result in a final remainder (see Year 3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process. The skill of calculating remainders should be secure before children are taught this method.

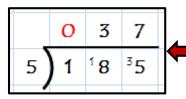


Step 2: Pupils move on to dividing numbers with up to 3 digits by a single digit, however problems and calculations provided should **not** result in a final answer with a remainder at this stage.

Children who exceed this expectation may progress

Children who exceed this expectation may progress to finding answers **with** remainders (see Year 5).

Important note:



When the answer for the first column is zero (as in the example), children should write the zero above to acknowledge its place, then must carry the number (1 in the example) over to the next digit as a remainder. This helps avoid mistakes when there is a 0 in the middle of the answer.

Real life problems need to be used routinely to help pupils gain a full
understanding and the ability to recognise the place of division and how
to apply it to problems. This should include money and measure
questions when the children are confident.

Key Vocabulary: share, share equally, one each, two each etc..., groups, groups of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor

Key skills for division at Y4:

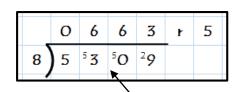
- Recall multiplication and division facts for all numbers up to 12 x 12
- Use place value, known and derived facts to multiply and divide mentally including multiplying and dividing by 10, 100 and 1
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a single-digit number
- Pupils practise mental methods and extend this to 3-digit numbers to derive facts, for example $200 \times 3 = 600 \times 600 \div 3 = 200$
- Pupils solve 2-step problems in contexts, choosing the appropriate operation and working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between ten children



Year 5 Divide up to 4-digit numbers by a single digit (including those with remainders)



Short division, including remainder answers:



The answer to $5309 \div 8$ could be expressed as $663\frac{5}{8}$, as $663 \cdot 5$, as 663.625 or rounded as appropriate to the problem.

Short division with remainders: Now that pupils have been introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it. This could be as a fraction, a decimal or as a rounded number depending on the context of the problem.

Money and measure contexts should be included regularly.

For children who are confident and accurate:

- The Year 6 method of short division to find a decimal answer can be taught so that remainders can be expressed as decimals.
- Long division can be introduced for pupils who are ready to divide by 2-digit numbers. See Year 6 for this method.

Key Vocabulary: share, share equally, one each, two each etc..., groups, groups of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime)

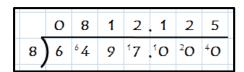
Key skills for division at Y5:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Year 4)
- Multiply and divide numbers mentally, drawing upon known facts
- Identify multiples and factors, including finding all factor pairs of a number and common factors of 2 numbers
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Use the vocabulary of prime numbers, prime factors and composite numbers (non-prime)
- Work out whether a number up to 100 is prime and recall prime numbers up to 19
- Divide numbers up to 4-digits by a single-digit number using the formal written method of short division and interpret remainders appropriately for the context
- · Use multiplication and division as inverses
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding
- Solve problems combining all four operations, including understanding of the equals sign, and including division for scaling by different fractions or problems involving simple rates



Year 6 Divide at least 4-digits by both single-digit and two-digit numbers (including decimals and quantities)

Short division for dividing by a single digit:



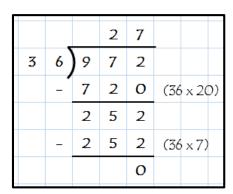
Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4-digits, and understand how to express remainders as fractions, decimals, whole number remainders or rounded numbers. Real life problem solving contexts need to be the starting point where pupils have to consider the most appropriate way to express the remainder.

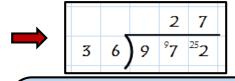
Calculating a decimal remainder: In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto the zero after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Dividing by 2-digit numbers:

Children should begin all division questions with a 2-digit divisor by writing down the times table for the divisor up to 10x. They should refer back to this throughout the calculation.

Children will initially be shown the long division method (chunking) but encouraged to progress to using the short division method for all calculations.





Children will use the times table they wrote down to find out "How many 36s go into 97?" then can either calculate the remainder mentally or with a quick written method at the side of the question. This method can be extended to give answers involving decimals if needed, however these calculation questions will generally have an integer answer.

<u>Key Vocabulary:</u> share, share equally, one each, two each etc..., groups, groups of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), common factor

Key skills for division at Y6:

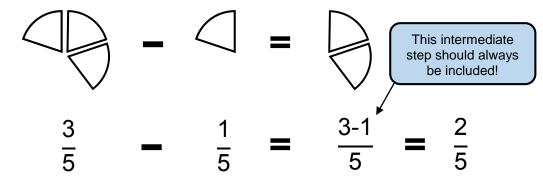
- Recall and use multiplication facts for all numbers up to 12x12 for complex calculations
- Divide numbers up to 4-digits by a two-digit number using the formal written method of long division and interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context. Use short division where appropriate
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Solve problems using all four operations
- Use estimation to check answers to calculations and determine accuracy in context
- Use written division methods in cases where the answer has up to 2 decimal places
- Solve problems which require answers to be rounded to a specified degree of accuracy

Year 3 Add and subtract fractions with the same denominator within one whole



Using a visual representation:

Children should be shown a variety of visual representations of adding and subtracting fractions, such as representing fractions as parts of a whole:



Or using a bar model:

$$\frac{4}{7}$$
 + $\frac{2}{7}$ = $\frac{4+2}{7}$ = $\frac{6}{7}$

As a fractions calculation:

Once secure using visual representations, children should progress to written fractions calculations, where the answer does not exceed one whole:

$$\frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$$

$$\frac{7}{10} - \frac{3}{10} = \frac{7 - 3}{10} = \frac{4}{10}$$

Children who are confident with these methods can now start to miss out the intermediate step and just write down the answer.

Real life problems should be included throughout to help develop a secure understanding, e.g.

"Jane eats $\frac{2}{5}$ of the pizza. Tom eats $\frac{1}{5}$ of the pizza. What fraction of the pizza have they eaten altogether?"

Key Vocabulary: numerator, denominator, out of

Key skills for fractions calculations at Y3:

- Understand that the denominator remains the same when adding and subtracting fractions with the same denominator
- Add and subtract single digit numbers confidently
- Solve word problems involving adding and subtracting fractions with the same denominator

Year 4 Add and subtract fractions with the same denominator (including mixed numbers)



Using a visual representation:

Children should be shown a clear visual representation where the answer is greater than one whole:

$$+ \frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

As in this example, children should convert their answer to a mixed number wherever possible. Children who are less confident should still include the intermediate step in the working out (see Year 3).

As a fractions calculation:

Once secure switching between mixed numbers and improper fractions, children should progress to written calculations, which may include mixed numbers. Initially, all the steps in the calculation should be shown but confident children can start to leave out intermediate steps (see progression below):

Addition:

$$\frac{3}{8} + \frac{7}{8} = \frac{3+7}{8} = \frac{10}{8} = 1\frac{2}{8}$$
$$\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = 1\frac{2}{8}$$
$$\frac{3}{8} + \frac{7}{8} = 1\frac{2}{8}$$

Subtraction:

$$1\frac{3}{8} - \frac{5}{8} = \frac{11}{8} - \frac{5}{8} = \frac{11-5}{8} = \frac{6}{8}$$
$$1\frac{3}{8} - \frac{5}{8} = \frac{11}{8} - \frac{5}{8} = \frac{6}{8}$$
$$1\frac{3}{8} - \frac{5}{8} = \frac{6}{8}$$

High ability children should be looking to see if their answers can be simplified, i.e. $1\frac{2}{8} = 1\frac{1}{4}$

Continue using word problems throughout!

Key Vocabulary: numerator, denominator, out of, mixed number, improper fraction

Key skills for fractions calculations at Y4:

- Convert between simple mixed numbers and improper fractions
- Recognise that fractions where the numerator is smaller than the denominator are less than
 one whole, equal to one whole if the numerator is equal to the denominator and greater
 than one whole if the numerator is bigger than the denominator
- Solve word problems involving adding and subtracting fractions with the same denominator, which may include mixed numbers and improper fractions

Year 5 Perform more advanced fractions calculations



Adding and subtracting fractions with related denominators:

Children should now be learning to add and subtract fractions where one denominator is a multiple of the other. The working out should be shown as below:

$$\frac{1}{3} + \frac{4}{9} = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$$

$$\frac{11}{16} - \frac{1}{4} = \frac{11}{16} - \frac{4}{16} = \frac{7}{16}$$

Children need to understand:

- Which denominator needs 'changing' and which stays the same
- In subtraction the order of the fractions must stay the same
- How to convert a fraction with one denominator into an equivalent fraction with a different denominator by multiplying both the numerator and the denominator by the same number

Whatever you do to the top, you do to the bottom!

Multiply proper fractions and mixed numbers by whole numbers:

Proper fractions multiplied by whole numbers:

$$\frac{2}{3} \times 5 = \frac{2 \times 5}{3} = \frac{10}{3} = 3\frac{1}{3}$$

Mixed numbers multiplied by whole numbers (two methods):

$$1\frac{1}{3} \times 7 = \frac{4}{3} \times 7 = \frac{4 \times 7}{3} = \frac{28}{3} = 9\frac{1}{3}$$
$$1\frac{1}{3} \times 7 = (1 \times 7) + (\frac{1}{3} \times 7) = 7 + \frac{7}{3} = 7 + 2\frac{1}{3} = 9\frac{1}{3}$$

Key Vocabulary: numerator, denominator, out of, mixed number, improper fraction, equivalent

Key skills for fractions calculations at Y5:

- Find equivalent fractions with given denominators
- Convert between mixed numbers and improper fractions
- Solve word problems involving addition and subtraction of fractions with related denominators including problems involving mixed numbers
- Solve word problems involving multiplying proper fractions and mixed numbers by whole numbers, giving answers as mixed numbers

Year 6 Perform fractions calculations using all four operations



Adding and subtracting fractions with different denominators:

$$\frac{3}{4} + \frac{5}{7} = \frac{3 \times 7}{28} + \frac{5 \times 4}{28} = \frac{21}{28} + \frac{20}{28} = \frac{41}{28} = 1\frac{13}{28}$$

Children should initially show all the above steps but as they get more confident can start to leave out intermediate steps (e.g. going straight to $\frac{21}{28}$ without the 3 x 7 stage).

Multiply simple pairs of proper fractions:

$$\frac{3}{8} \times \frac{2}{7} = \frac{3 \times 2}{8 \times 7} = \frac{6}{56} = \frac{3}{28}$$

Children should simplify their answers to these questions as much as possible.

Top times top, bottom times bottom!

Divide proper fractions by whole numbers:

$$\frac{4}{9} \div 6 = \frac{4}{9 \times 6} = \frac{4}{54} = \frac{2}{27}$$

Children should learn that dividing a fraction by a whole number means that the denominator should be multiplied by the whole number, then simplify the answer as much as possible.

Calculate decimal fraction equivalents:

5			0	, 6	2	5
8	=	8	5 ,	.⁵O	² O	⁴ O

Divide the numerator by the denominator using the short division method.

Key Vocabulary: numerator, denominator, out of, mixed number, improper fraction, equivalent

Key skills for fractions calculations at Y6:

- Find a common denominator for two fractions
- Find equivalent fractions with given denominators
- Solve word problems involving all four operations and fractions, including mixed numbers and improper fractions
- Simplify a fraction as far as possible by finding common factors of the numerator and denominator